N-22 SPECIFIC SURFACE OF CRACKED MEDIA AND ESTIMATION OF POROSITY BY SPLITTING OF SHEAR WAVES

B.P. SIBIRIAKOV
The Institute of Geophysics RAS (Sib. div.), Koptyg Avenue 3
Novosibirsk 630090, Russia

Summary. The product of specific surface σ_0, by average length l of a rectilinear segment of a fracture is linked with porosity f. It is shown that the above mentioned product, $\sigma_0 l$ is a main factor of decrease of velocities of P and S waves in fractured media. Because (at small value of porosity f) $\sqrt{f}$ is the quantity far from being small, even at small porosity (not a small specific surface of the fractures) elastic constants of fractured frameworks vary considerably in both of random fractures (isotropic situation) and of their predominant orientation (anisotropy). This circumstance gives us a possibility to calculate porosity as a function of elastic constants.

Introduction. The cracked medium has a very special geometry of pore space. In this medium may have small porosity but not small, even very large the specific surface. In order to describe the pore space geometry we need to use results of special mathematical discipline-integral geometry. There is a relation [1] between specific surface σ_0, average distance between two nearest crack l_0 and porosity f:

$$\sigma_0 l_0 = 4(1-f) \tag{1}$$

Evidently, the specific surface and not porosity itself is a parameter which is responsible for decreasing of P and S waves velocities, especially if this porosity is very small. On the internal surfaces boundary conditions are concentrated, which require the elimination of normal and tangent stresses (for dry cracks) and tangent stresses only (for cracks filled liquid). Just now there is a popular model of a cracked medium, considering isolated ellipsoids, and their aspect ratio is very important parameter of pore space [2]. In this paper author tries to prove that there is a possibility to estimate elastic parameters in isotropic and anisotropic media by using of specific surface, porosity and Poisson ratio of matrix only.
In order to represent a relation between displacement and stresses in the arbitrary complicate microinhomogeneous body we use integral equation, which created by Kupradze [3] in the form:

$$\frac{1}{2}u_k(x) = \frac{1}{4\pi}\left[\int_S N_i(y)\Gamma_{ik}(x,y)dS_y - \int_{S+\sigma} u_i(y)N_i\Gamma^k(x,y)dS_y\right]. \tag{2}$$

In the formula (2) u_k is a displacement vector, $\Gamma_{ik}(x,y)$-Green-tensor of elastic matrix, $N_i(y)$-pseudo stresses, which created by Kupradze, $N_i\Gamma_{ik}(x,y)$ –pseudo stresses from Green-tensor.
Symbol S means an external surface, while σ is the internal surface. Pseudo stresses are acting in some artificial medium with elastic constants

$$\lambda^* = \frac{(\lambda+\mu)(\lambda+2\mu)}{\lambda+3\mu}; \mu^* = \frac{\mu(\lambda+\mu)}{\lambda+3\mu} \tag{3}$$

and λ,μ are usual Lame constants for real elastic medium.
Method of field calculation. The main problem of integral equation (2) solution is a complicate internal surface in the second integral (2). A variable point y located on the external surface (simple) and internal one (complicate). In order to calculate a displacement field it is reasonable to divide a contribution of integral

$$\int_\sigma u_i(y)N_i\ \Gamma^k(x,y)dS_y \tag{4}$$

into two parts, namely: near acting zone and long distance acting zone. Near distance zone includes point y, which is located on the two levels of the same plane section of crack. Long distance zone of acting is determined as a set of points y, which are located on the crack planes, excepting the plane, containing point x. In the near acting zone there is two cases (Fig.1).

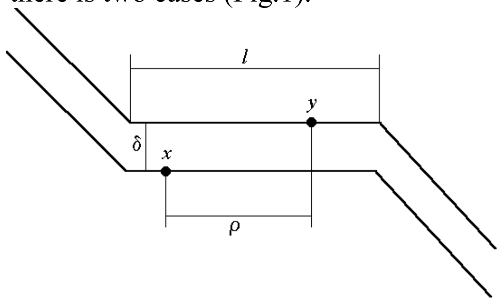

Fig.1.

If points x, y are located on the same plane (level), a contribution to the integral (6) is equal to zero, because $\frac{\partial}{\partial n}(\frac{1}{r}) = -\frac{Cos(r,n)}{r^2} = 0$.

If points x, y are located on the opposite levels of the same crack plane section, the integral (6) can be calculated very simply. Really, in this case $Cos(r,n) = \frac{\delta}{\sqrt{\delta^2+\rho^2}}$, and evidently,

$\frac{Cos(r,n)}{r^2} = \frac{\delta}{(\delta^2+\rho^2)^{3/2}}$, where δ is an average thickness of crack and ρ the projection of distance between points x, y to the plane of crack. Using the field $u_i^0(x)$ as a field in solid body without cracks (simple field) we can write mentioned contribution due to near acting zone by expression:

$$u_i(x) = (1 - \frac{2\delta}{\sqrt{l^2+4\delta^2}})[u_i^0(x) + \delta\frac{\partial u_i^0}{\partial n}(x)] + \delta\frac{\partial u_i^0}{\partial \rho}(x)(\ln\frac{\sqrt{l^2+4\delta^2}}{2\delta} - \frac{l}{\sqrt{l^2+4\delta^2}}) \quad (5)$$

In the formula (7) the main effect of a pore space is giving by factor $\frac{2\delta}{\sqrt{l^2+4\delta^2}} \approx \frac{2\delta}{l}$, where l is a length of an average straight segment of crack.

A long distance zone of acting gives less contribution in the displacement field due to special dipole effect between two points y_1, y_2 which are located on opposite sides of crack levels (Fig.2).

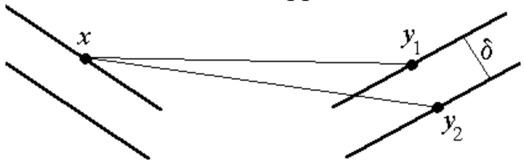

Fig.2.

In fact there are two normal vectors in points y_1, y_2 with opposite signs. It causes a differentiating of the field with multiply factor δ. In the paper [4] it was shown that such effect is proportional to porosity f, and for small porosity this effect very small compared to $\frac{2\delta}{\sqrt{l^2+4\delta^2}} \approx \frac{2\delta}{l}$. The common effect of long distance action may be expressed by the integral equation with regular kernel $M_{ik}(x,y)$ in the form:

$$\frac{1}{2}u_i(x) + \frac{\sigma_0}{3}\frac{1}{4\pi}\int_V u_i(y)M_{ik}(x,y)dV_y = R(x), \quad (6)$$

where area of integration is a volume V, not the complicated surface σ, and the free term $R(x)$ is a value, which is proportional to δ or porosity f, namely:

$$R_i(x) = \frac{\delta}{2}[\frac{\partial u_i^0}{\partial x_l} n_l + (\ln\frac{l+\sqrt{l^2+4\delta^2}}{2\delta} - \frac{l}{\sqrt{l^2+4\delta^2}})\frac{\partial u_i^0}{\partial x_l}\sqrt{1-n_l^2} + \frac{2\xi_i}{\sqrt{l^2+4\delta^2}}]. \quad (7)$$

In expression (7) vector ξ_i is the value of an order l, and for rocks with a small porosity but not small specific surface an effect of long distance interactions is negligibly small.

Calculation of elastic constants. The idea of calculation of elastic constants just now is very simple. Really, taking into account that the main effect for the field is given by near acting zone, we can calculate the deficit of potential energy due to cracks. This energy is a product of stresses in loaded body without cracks by displacement, which results from elimination of forces acting on the crack surface. Let's examine some cube loaded in z-direction and with rigid planes perpendicular to axes x and y. It potential energy is $E_0=\frac{1}{2}(\lambda+2\mu)e_0^2$ where e_0 is a strain. Let's assume that orientation of crack plane section is given by directed cosines n_x, n_y, n_z with evident relation: $\sqrt{n_x^2+n_y^2+n_z^2}=1$. The normal and tangent forces acting on the crack surface are giving by expression:

$$P_n = \sigma_{ik} n_i n_k, \quad P_\tau = \sqrt{P^2 - P_n^2}, \quad P^2 = P_i^2, \quad P_i = \sigma_{ik} n_k. \quad (8)$$

In the formula (8) P_n P_τ are normal and tangent forces. These forces are expressed from elementary stresses for cube without cracks, namely:

$$\sigma_{zz} = (\lambda + 2\mu)e_0 \qquad \sigma_{xx} = \sigma_{yy} = \lambda e_0 \quad (9)$$

As to the displacement vector, it may be expressed (for small thickness of crack δ compared to average straight segment of crack l) in the form of Green-tensor for half space. It gives us simple formulas for deficit of normal and tangent parts of energy due to cracks:

The total deficit of a potential energy due to cracks is:

$$E = \frac{\sigma_0 l}{4\mu}[(1-\nu/2)P^2 - \nu/2P_{n0}^2] \quad (10)$$

In the formula (10) there is a dimensionless product $\sigma_0 l$ of specific surface by average straight segment of crack, and ν -the Poisson ratio. We can declare that the difference between initial elastic energy E_0 and total elastic energy E gives us the potential energy for equivalent anisotropic body (if cracks have predominant orientation) or isotropic cracked body (if cracks have random orientation). The main relation is:

$$E_0 - E = W = \frac{1}{2}C_{11}(e_{xx}^2 + e_{yy}^2) + (C_{11} - 2C_{66})e_{xx}e_{yy} + \frac{1}{2}C_{33}e_{zz}^2 + C_{13}e_{zz}(e_{xx}+e_{yy}) +$$
$$\frac{1}{2}C_{44}(e_{xz}^2 + e_{yz}^2) + \frac{1}{2}C_{66}e_{xy}^2.$$

(11)

An expression (11) is valid for transversely isotropic body. If there is one strain e_{zz} only the potential energy (11) takes a simple form; $W=1/2C_{33}e_{zz}^2$,while the deficit of potential energy due to cracks is:

$$E = P^2\frac{\sigma_0 l}{4\mu}[(1-\nu/2)(\sigma_{xx}n_x + \sigma_{yy}n_y + \sigma_{zz}n_z)^2 - \nu/2(\sigma_{xx}n_x^2 + \sigma_{yy}n_y^2 + \sigma_{zz}n_z^2)^2] =$$
$$= P^2\frac{\sigma_0 l}{4\mu}[(1-\nu/2)((1-\gamma^2)(n_x^2+n_y^2)+n_z^2)) - \nu/2((1-2\gamma^2)^2(n_x^2+n_y^2)^2 + n_z^4$$
$$+ 2(1-2\gamma^2)n_z^2(1-n_z^2))]$$

(12)

In (12) γ is a ratio V_S/V_P . Formula for elastic constant C_{33} takes a form:

$$\frac{C_{33}}{\lambda+2\mu} = 1 - \frac{\sigma_0 l}{2\gamma^2}[(1-\nu/2)(n_z^2 + \beta^2(1-n_z^2)) - \nu/2(\beta^2(n_x^2+n_y^2)^2 + n_z^2$$
$$-2\beta n_z^2(1-n_z^2))] \quad (13)$$

The value $\beta=1-2\gamma^2$. The analogous formula for constant C_{11} is given by expression looks like (13) where n_x and n_z are change their places. For chaotic orientation of cracks $<n_x^2>=1/3$, $<n_x^4>=1/5$, $<n_x^2 n_y^2>=1/15$. Such values determine elastic constants for isotropic media. As to value C_{13} this constant can not be determined by the mentioned mind experiment. We can put $C_{13}=<C_{33}>-2<C_{66}>$, i.e. by average constants for isotropic body. The determination of constants C_{44}, C_{66} requires mind experiments with shear strains. For cube loaded by shear stress σ_{xz} only, a total energy, which appeared due to cracks is giving by expression:

$$E=\frac{\sigma_0 l}{2\mu}[(1-\nu/2)\sigma_{xz}^2 n_z^2-\nu/2\sigma_{xz}^2 n_x^2 n_z^2]=\frac{\sigma_0 l}{2\mu}\sigma_{xz}^2 n_z^2(1-\nu/2-\nu/2n_x^2). \tag{14}$$

Taking into account (14) we can write a following expression for C_{44}, namely:

$$C_{44}=\mu[1-\frac{\sigma_0 l}{4}n_z^2(1-\nu/2-\nu/2n_x^2)]. \tag{15}$$

Changing n_z into n_y we can write an analogous expression for C_{66} in the form

$$C_{66}=\mu[1-\frac{\sigma_0 l}{4}n_y^2(1-\nu/2-\nu/2n_x^2)]. \tag{16}$$

Dimensionless product $\sigma_0 l$ and porosity. In order to determine the average straight length of a segment of crack l we can examine a part of volume V with characteristic dimension l_0, according to formula (1). Let's examine a sphere with radius $l_0/2$ (with volume $4\pi/3(l_0/2)^3$) and surrounding pore space with volume $f4\pi/3(l_0/2)^3$), Fig.3.

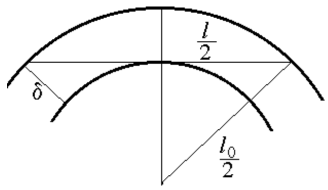

Fig.3.

As a length l we can choose a distance within the spherical layer. This value is easy to calculate:

$$l=l_0\sqrt{2/3f} \tag{17}$$

and dimensionless product $\sigma_0 l$ depends on the square root of porosity only:

$$\sigma_0 l=4(1-f)\sqrt{2/3f} \tag{18}$$

The ratio of constants

$$\frac{C_{44}}{C_{66}}=\frac{1-(1-f)\sqrt{2/3f}\,n_z^2(1-\nu/2-\nu/2n_x^2)}{1-(1-f)\sqrt{2/3f}\,n_y^2(1-\nu/2-\nu/2n_x^2)}. \tag{19}$$

Constants $C_{44}=\rho V_{S1}^2$ and $C_{66}=\rho V_{S2}^2$ can be measured by VSP investigations in the wells. One of them is fast shear wave and other is slow shear wave. Measuring such two velocities we can estimate the porosity of cracked medium with predominant orientation of cracks, because this parameter almost doesn't depend on material of matrix. This method requires the Poisson ratio of matrix only, but this value for carbonate rocks is near to *1/4*.

References.

1. F.A. Usmanov. Principles of Mathematical Analysis of Geological Structures.*[In Russian], Tashkent, FAZ UzSSR,120-124.*

2. J. Douma. Crack-included Anisotropy and its Effect on Vertical Seismic Profiling. *Geologica Ultraiectina, 1988, N54, Utrecht, 167p.*

3. V.D. Kupradze. Potential Methods in Elasticity. *[In Russian] Moscow, 1963, 472p.*

4. B.P. Sibiriakov. Structure of Pore Space and Elastic Properties of Small Porous Cracked Media. *Geologia i Geofizika (Russian Geology and Geophysics), 1996,vol.37, N9, p146-155.*